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METHOD FOR OBTAINING WET-BULB TEMPERATURES  
BY MODIFYING THE PSYCHROMETRIC FORMULA

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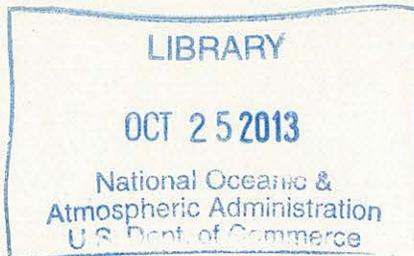


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METHOD FOR OBTAINING WET-BULB TEMPERATURES BY MODIFYING  
THE PSYCHROMETRIC FORMULA

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Abstract. In computer processing of shipboard data obtained during the Barbados Oceanographic and Meteorological Experiment (BOMEX), conducted in 1969, values of the wet-bulb temperature had to be obtained from the known dry-bulb temperatures and relative humidities. This cannot be done by the usual algebraic process of solving a polynomial equation, and an algorithm was therefore developed to approximate the psychrometric formula by a polynomial equation. For the BOMEX data, the algorithm gave wet-bulb temperatures that were consistent within  $\pm 0.01^{\circ}\text{C}$  with the input values of dry-bulb temperature, relative humidity, and pressure. The algorithm is also accurate for a wider range of meteorological variables, and can be applied to other nonpolynomial equations.

## 1. INTRODUCTION

During the Barbados Oceanographic and Meteorological Experiment (BOMEX), conducted in 1969, relative humidities were measured aboard each of the five research ships by both a psychrometer and a lithium chloride hygrometer. In the computer program used for generating relative humidity values, the psychrometer dry- and wet-bulb temperatures were used when available. In instances where the wet bulb dried up, or measurements were improperly recorded or omitted, good estimates of the wet-bulb temperature could still be obtained by using the dry-bulb temperatures and relative humidities recorded by the lithium chloride hygrometer. The method for obtaining these estimates is discussed in this memorandum.

## 2. PSYCHROMETRIC FORMULA

Measurement of the wet-bulb temperature by a psychrometer is obtained when a quasi-equilibrium is reached between the heat lost through evaporation and the heat gained from the surrounding air. If we assume that a true equilibrium holds and that the ambient air is essentially unchanged by the evaporated water, a wet-bulb temperature can be defined by means of the equations

of thermodynamics (Smithsonian Institution 1951, p. 348). How closely the physical process assumed in this definition is actually reflected by the psychrometer measurement is still not fully known (Bindon 1965). The theoretical description, however, suggests the form the wet-bulb equation should have, and actual measurements can then fix the values of empirical parameters in this equation. Ferrel (Smithsonian Institution 1951, p. 365), in summarizing the results of his experiments, gives the following formula as representative of an adequately ventilated psychrometer:

$$E_s(T_w) - E(T_D) = 0.00066 * P * (1 + 0.00115 * T_w) * (T_D - T_w), \quad (1)$$

where

$E(T)$  = vapor pressure of water at temperature  $T$ , and the subscript  $s$  denotes the saturation value, in mb.

$T_w$  = true wet-bulb temperature, in  $^{\circ}\text{C}$ .

$T_D$  = dry-bulb temperature, in  $^{\circ}\text{C}$ .

$P$  = ambient pressure, in mb.

Vapor pressures are measured over water. For wet-bulb temperatures below freezing there is a question of whether vapor measurements over ice would have greater validity. Since the BOMEX sea-level data were obtained under tropical conditions, this question need not be considered. The temperatures during the experiment ranged from 20 to 30 $^{\circ}\text{C}$ , and the relative humidities from 60 to 90 percent.

Relative humidity is brought into eq (1) by use of the definition

$$\hat{E}(T) = (\text{RH}/100) * E_s(T),$$

where

RH = relative humidity, in percent.

This changes eq (1) to the form

$$E_s(T_w) - (\text{RH}/100) * E_s(T_D) = 0.00066 * P * (1 + 0.00115 * T_w) * (T_D - T_w). \quad (1a)$$

### 3. METHOD FOR MODIFYING THE PSYCHROMETRIC FORMULA

#### 3.1 General Approach

Equation (1a) can be evaluated qualitatively by examining the case where the wet-bulb depression  $\Delta T \equiv T_D - T_W$  is small. For depressions of less than  $2^\circ\text{C}$ , the saturation vapor pressure is approximately linear, i.e.,

$$E_s(T_W) = E_s(T_D - \Delta T) \approx E_s(T_D) - b \cdot \Delta T,$$

where

$b$  = slope of saturation vapor curve in the region of  $T_D$ .

Using this approximation in eq (1a) and neglecting the term  $0.00115 \cdot T_W$ , which is small in comparison with 1.0, gives

$$\Delta T \approx \frac{[1 - (\text{RH}/100)] \cdot E_s(T_D)}{b + 0.00066 \cdot P}. \quad (2)$$

From this we see that the wet-bulb depression  $\Delta T$

- (a) increases with increasing  $T_D$  (through  $E_s(T_D)$ ),
- (b) decreases with increasing RH, and
- (c) decreases with increasing  $P$ .

To solve eq (1a) accurately for  $T_W$  is far more difficult. The reason is that the saturation vapor pressure is an exponentially increasing function of temperature, i.e.,

$$E_s(T) = 6.1078 \cdot e^{f(T)},$$

where

6.1078 = saturation vapor pressure, in mb, at  $0^\circ\text{C}$ ,

$e = 2.718$  is the base for natural logarithms, and

$f(T)$  = a monotonically increasing function of  $T$ .

When this expression for  $E_s(T)$  is substituted into eq (1a), the resulting equation can no longer be solved for  $T_W$  by the usual algebraic process of finding the roots of a polynomial equation.

When the exponential function (app. B) is expanded into a Taylor series, the result is

$$e^{f(T+\delta T)} = e^{f(T)} * [1 + a_1(\delta T) + a_2(\delta T)^2 + \dots] \quad (3)$$

In other words, the function factors into an exponential times a polynomial, or, stated in another way,

$$E_s(T+\delta T) = E_s(T) * [\text{a polynomial in the small temperature difference } \delta T]. \quad (3a)$$

The smaller the temperature difference  $\delta T$ , the fewer powers will have to be retained in the polynomial to obtain the desired accuracy.

With the above in mind, our approach was to:

- (a) Use the psychrometric formula in the usual way to derive RH's for a range of  $T_w$ 's and  $T_D$ 's and a fixed pressure near sea level.
- (b) Invert the process and use Marvin's (1941) tabulation of  $T_w$  as a function of  $T_D$  and RH for the fixed pressure.
- (c) Find a simple formula that, given  $T_D$ , RH and P, would yield a reasonably accurate first approximation,  $T_w^{(o)}$ , of  $T_w$ .
- (d) Use the factor property of eq (3) to write  $E_s(T_w)$  as a polynomial in the temperature difference  $\delta T \equiv T_w - T_w^{(o)}$ . Using only the first three terms of the polynomial changes the psychrometric formula into a quadratic equation, in terms of  $\delta T$ , that can be solved.
- (e) Add  $\delta T$  to  $T_w^{(o)}$  to obtain  $T_w$ .

### 3.2 Saturation Vapor Pressure Formula

To express the saturation vapor pressure analytically, we chose, for reasons of simplicity, Tetens' formula (Berry et al. 1945, p. 343),

$$E_s(T) = 6.1078 * 10^{\left(\frac{7.5 * T}{237.3 + T}\right)} \approx 6.1078 * e^{\left(\frac{17.27 * T}{237.3 + T}\right)} \quad (4)$$

where T is in  $^{\circ}\text{C}$ . For the range of surface temperatures in the BOMEX data, this formula and the Smithsonian Meteorological Tables (Smithsonian Institution 1951, p. 350) agree within  $\pm 0.01^{\circ}\text{C}$  for the same saturation vapor pressure.

Based on Tetens' formula, the quadratic in  $\delta T$  becomes

$$[\text{Quadratic in } \delta T] = 1 + a_o * \delta T + \left[ \left( \frac{a_o^2}{2} - \frac{a_o}{237.3 + T_w^{(o)}} \right) * (\delta T)^2 \right], \quad (5)$$

where

$$a_o \equiv \frac{17.27 * 237.3}{[237.3 + T_w^{(o)}]^2}$$

An estimate of the consistency between eq (3) and eq (4), when only three terms of the polynomial are kept, can be gained by letting  $\delta T = 1^\circ\text{C}$ . This, as the next section shows, is a reasonable value. Then, for  $T_w = 21^\circ\text{C}$  and  $T_w^{(o)} = 20^\circ\text{C}$ ,

$$E_s(T_w) = 24.8769, \text{ and}$$

$$E_s(T_w^{(o)}) * [\text{Quadratic in } \delta T] = 24.8763.$$

### 3.3 First Approximation Formula

To derive a simple formula for computing  $T_w^{(o)}$ , the first approximation of the wet-bulb temperature, we used Marvin's Psychrometric Tables (1941), which give  $\Delta T$  for a range of  $T_D$  and RH values, and a set pressure of 982 mb. We then adjusted the depressions for a pressure of 1,000 mb and constructed table 1. A least-squares fit was run, giving the depression as a function of  $T_D$ , RH, and  $T_D * RH$ . The least-squares coefficients were rounded off for simplicity and to make the fit accurate, on the average, to within  $\pm 1^\circ\text{C}$  or better. This resulted in the formula,

$$[T_D - T_w^{(o)}]_{1000} = 6.6 + 2 * \left( \frac{T_D - 20}{10} \right) - 3 * \left( \frac{RH - 50}{20} \right) - 1 * \left( \frac{T_D - 20}{10} \right) * \left( \frac{RH - 50}{20} \right). \quad (6)$$

The depression increases as the ambient pressure decreases. A table similar to table 1 was therefore constructed for  $P = 500$  mb, and it was found that for a fixed RH the differences between the depressions at the two pressure levels were fairly independent of  $T_D$ . Based on this observation, the following formula was obtained for depressions at any pressure up to 500 mb:

$$[T_D - T_w^{(o)}]_P = [T_D - T_w^{(o)}]_{1000} + \left( \frac{1000 - P}{500} \right) * \left[ 1.4 - 0.9 * \left( \frac{RH - 50}{20} \right) + 0.15 * \left( \frac{RH - 50}{20} \right)^2 \right]. \quad (6a)$$

Table 1.--Wet-bulb depression,  $\Delta T$ , as a function of dry-bulb temperature and relative humidity at a pressure of 1,000 mb

Dry-bulb temperature in °C	Relative humidity in percent				
	90	70	50	30	10
40	1.7	5.4	9.7	14.8	21.3
30	1.4	4.5	8.0	12.0	16.7
20	1.1	3.6	6.2	9.1	12.4
10	0.8	2.6	4.5	6.4	8.5

This formula was checked at pressures of 1,015 mb and 750 mb also, and proved to be an adequate first approximation of the depression, and hence of  $T_w$ .

Appendix A lists the steps used in the complete calculation of the wet-bulb temperature. These steps can easily be incorporated into a computer program. Some of the numerical values may change if a different function for the saturation vapor pressure is used, but the general procedure remains the same.

#### 4. RESULTS

As a test, the method was applied to data representative of BOMEX sea-surface data. The pressure was fixed at 1,015 mb, the dry-bulb temperatures ranged from 20°C to 30°C, and the relative humidities from 60 to 90 percent. The results are shown in table 2. The numerical agreement of the fourth and fifth columns in this table, which are the left and right sides of eq (1a), illustrates the accuracy of the method. From further tests, it seems reasonable to conclude that, on the average, the method will assign a wet-bulb temperature that is consistent to within  $\pm 0.01^\circ\text{C}$  accuracy with the following ranges of  $T_D$ , RH, and P input data:

Wet-bulb temperatures from 0°C to 40°C.

Relative humidities greater than 10 percent.

Pressures greater than 500 mb.

The method can also be extended to wet-bulb temperatures below freezing once it is determined, based on physical conditions, whether the saturation vapor pressure should be taken over water or over ice.

Table 2.--Accuracy of the method, shown by numerical agreement between columns 4 and 5, for a range of dry-bulb temperatures and relative humidities and a pressure of 1,015 mb ( $T_D - T_w$  is rounded off)

$T_D$ (°C)	RH (percent)	$T_D - T_w$ (°C)	$E_s(T_w) - E(T_D)$ (mb)	$0.00066 * P * (1 + 0.00115 * T_w) * (T_D - T_w)$ (mb)
30	90	1.395	0.96485	0.965108
30	75	3.641	2.51286	2.51286
30	60	6.102	4.20012	4.20005
25	90	1.260	0.866932	0.867138
25	75	3.272	2.24666	2.24666
25	60	5.450	3.73335	3.73332
20	90	1.120	0.766229	0.766314
20	75	2.891	1.9749	1.9749
20	60	4.785	3.26187	3.26186

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- Smithsonian Institution, Smithsonian Meteorological Tables, Sixth Revised Edition, Section VII, Publication 4014, City of Washington, 1951, pp. 347-385.

## APPENDIX A

Method for solving the psychrometric formula, eq (1a), for  $T_w$ , given  $T_D$ , RH, and P:

- (1) Compute  $T_w^{(o)}$  by first computing  $[T_D - T_w^{(o)}]_{1000}$  from eq (6) and using this value in eq (6a).
- (2) Compute  $EO = E_s(T_w^{(o)})$ ,  $E = (RH/100) * E_s(T_D)$  using eq (4).
- (3)  $AO = 17.27 * 237.3 / (237.3 + T_w^{(o)})^2$   
 $F = 0.00066 * P / EO$
- (4)  $CO = F * (1 + 0.00115 * T_w^{(o)}) * (T_D - T_w^{(o)}) + E / EO$   
 $C1 = F * [0.00115 * (T_D - T_w^{(o)}) - (1 + 0.00115 * T_w^{(o)})]$   
 $C2 = F * 0.00115$
- (5)  $A = [AO^2 / 2] - [AO / (237.3 + T_w^{(o)})] + C2$   
 $B = AO - C1$   
 $C = 1 - CO$
- (6)  $\delta T = [-B + \sqrt{B^2 - 4 * A * C}] / 2 * A$
- (7)  $T_w = T_w^{(o)} + \delta T$

## APPENDIX B

By adding and subtracting  $f(T)$  in the argument of the exponential function, we can write

$$e^{f(T + \delta T)} = e^{f(T)} + [f(T + \delta T) - f(T)]. \quad (1)$$

Then, using the addition property  $e^{x+y} = e^x * e^y$ , we can rewrite the result in the form

$$e^{f(T + \delta T)} = e^{f(T)} * e^{f(T + \delta T) - f(T)}. \quad (2)$$

The second factor on the right of eq (2) is a function of both  $T$  and the variable  $\delta T$ . It can be expanded into a power series in the variable  $\delta T$  to finally give

$$e^{f(T + \delta T)} = e^{f(T)} * [1 + a_1(\delta T) + a_2(\delta T)^2 + \dots].$$